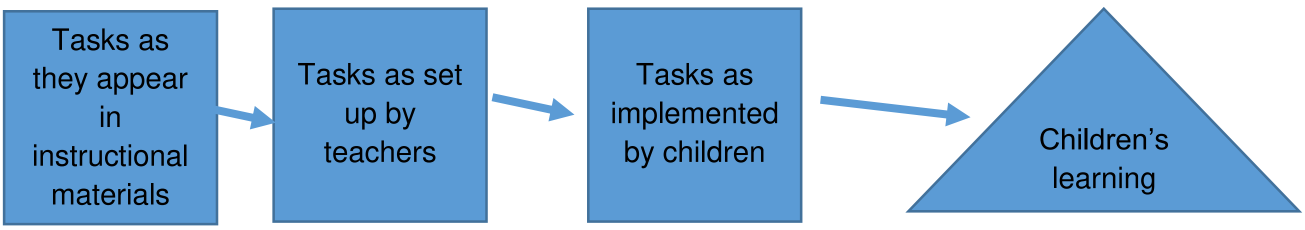
Thinking about tasks

The tasks we present to children shape the development of their mathematical thinking, their understanding of what mathematics is and their disposition toward the subject. Tasks are chosen to meet the goals of a lesson. A teacher chooses a task to focus students' attention on a particular mathematical ideas or content, but opportunities for students to experience processes such as connecting, communicating, reasoning, argumentation, justifying, representing, problem-solving, and generalising are also very important. These processes are central to mathematization where children are supported to interpret and express their everyday experiences in mathematical form and analyse real world problems in a mathematical way (NCCA Research Report 17). It is also important to realise that children can learn mathematical content through problem-solving activities. In fact, we remove a lot of the challenge from problems if we pre-teach content or procedures. Cognitively-demanding or challenging tasks are seen to be important for developing and extending thinking. A well-chosen task will be pitched just beyond current levels of understanding so that it is achievable with some effort. For more information see Chapter 2 of NCCA Research Report 18.

Tasks as they appear in textbooks, instructional materials or plans often change in nature as they are enacted in the classroom. Seminal research by Stein and Smith (1998) has identified a number of phases through which a task passes. For example, a teacher's goals and knowledge of the children in his/her class may influence how he/she decides to set up the task. Similarly, classroom norms and children's learning habits and dispositions will influence how children implement tasks. Research has shown that when teachers try to introduce problem-solving activities, they sometimes provide too much scaffolding and remove the cognitive challenge from their students.



The Mathematics Tasks Framework (Stein & Smith, 1998)

Stein, M, K., & Smith, M. S., (1998). Mathematical tasks as a framework for reflection: From research to practice. *Mathematics teaching in the middle school*, 3(4), 268-275

Tasks as they appear in instructional materials or plans

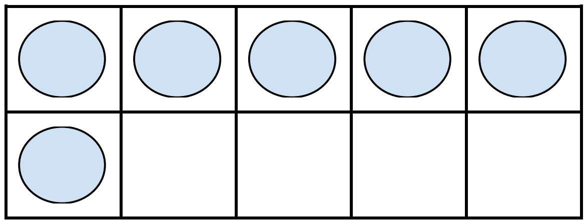
There are a number of ways to think about tasks. An **'open' task** is one which either has more than one possible solution or more than one possible solution method. For example, rather than directing children to sort toys in a certain way, children can be challenged to find their own ways to sort collections (see Infant activity plan on Sorting). Such tasks offer more possibilities for success and for distinctive individual methods than traditional approaches to mathematics teaching where a teacher demonstrates a method and children copy it. It is also possible to consider if different levels of challenge are possible for children as they engage in tasks. For example, **'low threshold, high ceiling'** tasks are ones which all children can get started on, but which also offer opportunities for rich and challenging mathematics thinking. An example used in our videos is the number problem for Senior Infants/First Class:

> *The principal is expecting a delivery of ten boxes. She will stack some in her office and some in the secretary's office. How many different ways can she stack the boxes?*

All children should be able to begin this problem by listing combinations of number bonds to ten. Children who find this aspect easy will be challenged by considering (and systematically recording) the different combinations. For information on 'low threshold, high ceiling' tasks, see this short article on Nrich.

Another way to think about the **cognitive challenge** of mathematics tasks has been presented by Stein and Smith (1998). They suggest that tasks can be split into those with lower- level cognitive demands and those with higher-level demands. Tasks with lower-levels of cognitive challenge might involve memorization or procedures without connections. Tasks with higher-level demands might involve procedure with connections or activities concerned with 'doing mathematics'. For such tasks, the solution is not obvious and the task cannot be solved using previously practiced routines or procedures. Children must analyse the task and explore mathematical concepts, processes or relationships.

They will also need to monitor their thinking and strategically choose their actions as they attempt to complete the task.

Stein and Smith's (1998) framework for analysing tasks	
Lower-level demands	**Higher-level demands**
Memorization Tasks involve either memorizing facts and rules or reproducing previously learned facts and rules. Procedures cannot be used because none exist or because the time allowed is too short. The requirements of the task are very clearly stated. Tasks have no connection to the concepts that underlie the facts and rules to be memorized/practiced. *Example:* *What number do I need to add to 6 to get 10? (Immediate answer expected from memory).*	Procedures with Connections Tasks focus attention on the use of procedures with the aim of developing conceptual understanding. Pathways suggested by the task involve broad general procedures that have close connections to underlying conceptual ideas (in contrast with limited algorithms with no connection to concepts). These tasks are often represented in multiple ways (e.g., diagrams, manipulatives, symbols, or problem contexts). Meaning is developed from making connections across representations. The tasks require some cognitive effort. Students must engage with the concepts as well as the procedure to complete the task successfully. Example:  *Picture what six would look like on the ten frame. Now can you figure out how many more we would need to make 10?* *(While this task may be solved with a procedure, i.e. counting the empty boxes, the representation means that this is closely linked to the underlying concept of combining numbers to make 10 and number partitions more generally - six being 'made up of' five and one might be noticed from the ten frame).*
Procedures without Connections Tasks are algorithmic, of limited cognitive demand and focused on producing correct answers instead of developing understanding. It is obvious which procedure is needed to solve the task (e.g. from prior instruction or experience). There are no connections to the concepts underlying the procedure. If explanations are required, they only focus on describing the procedure that was used. Example: *6 + ___ = 10* *Use your number line to figure out the answer.* *(This is seen a procedure without connections when the number line is used as an aid only. The child will say 'one, two, three, four' as they tap the numbers 7, 8, 9 10 on the number line but without further input from the teacher the concepts are unclear, e.g. how counting on relates to addition or how what is represented on the number line relates to the counting that the child undertakes.*	Doing Mathematics These tasks require considerable cognitive effort and cannot be solved using previously practiced routines or procedures. The solution requires complex, non-algorithmic thinking where children must analyse the task and explore mathematical concepts, processes or relationships. The tasks require self-monitoring and self-regulation, i.e. students will need to monitor their thinking and strategically choose their actions as they attempt to complete the task. The unpredictable nature of the solution process may be challenging for some students. *Example:* *The principal is expecting a delivery of ten boxes. She will stack some in her office and some in the secretary's office. How many different ways can she stack the boxes?*

Thinking about tasks as set up by teachers and implemented by children
Teaching using cognitively-demanding tasks is challenging for teachers. Careful attention is needed to the mathematics of the task and also to children's levels of understanding and ways of working. Teachers must provide support where it is needed but also avoid removing all challenge from the children. Research has shown that sometimes teachers remove the 'problem' from the problem-solving activity. This happens when a teacher 'takes over' and tells the children exactly what to do rather than allowing children to grapple with the problem and come up with their own ideas.

The research of Stein and Smith (1998) suggests that sometimes cognitively-challenging tasks do not result in children's higher-order thinking because of timing issues. Too little time may prevent meaningful engagement with mathematics while too much time can lead to off-task or unfocused behaviour. Tasks are less likely to be successful if they do not connect with interests, prior knowledge, or previous experience of children. Finally, tasks must be enacted in such a way as to make children accountable for their mathematical activity, e.g. clear expectations about completeness and exactness of explanations and reasoning.

Factors associated with children's persistence in attempting challenging tasks:

- Tasks that build on children's' prior knowledge
- Scaffolding (simplifying the task in some way without removing complexity, e.g. same task within a smaller number range)
- Demonstration of high-level performance by the teacher and/or capable students
- Making conceptual connections
- Appropriate amounts of time to explore ideas and make connections
- Encouragement of student self-monitoring
- A sustained press for explanation, meaning, and understanding by the teacher or other students (Stein & Smith, 1998).

Some of these issues are addressed more fully in the section on Talk. Other useful ideas can be found in the professional development module on problem-solving produced by the Mathematics Assessment Resource Service and Shell Centre. This material is aimed at those teaching upper primary and secondary school, but some of the general messages are still relevant. In particular, the authors discuss the notion of scaffolding. The key idea, common to Montessori approaches, is that a teacher should encourage children to do as much as they are capable of independently and only provide the minimum of support needed to help them succeed. Scaffolding should be removed once the child begins to cope; otherwise the child may become dependent on it and never reach independent mastery. An adapted version of their practical advice for teaching problem-solving is included below.

Practical Advice for Teaching Problem Solving	
Allow students time to understand and engage with the problem Discourage students from rushing in too quickly or from asking you to help too soon.	• *Take your time, don't rush.* • *What do you know?* • *What are you trying to do?* • *Don't ask for help too quickly - try to think it out between you.*
Offer strategic rather than technical hints Avoid simplifying problems for students by breaking it down into steps.	• *How could you get started on this problem?* • *What have you tried so far?* • *Could you draw a picture or use materials to help you figure this out?* • *Is there a good number/shape/ example you can try out?* • *Is there a good order to try/draw/write our ideas?*
Encourage students to consider alternative methods and approaches Encourage children to compare their own methods.	• *Is there another way of doing this?* • *Describe your method to the rest of the group* • *Which of these two methods do you prefer and why?*
Encourage explanation Make children do the reasoning, and encourage them to explain to one another.	• *Can you explain your method?* • *Can you explain that again differently?* • *Can you put what Sarah just said into your own words?* • *Can you write that down/draw a picture or use materials to show what you have done?*
Model thinking and powerful methods When children have done all they can, they will learn from being shown a powerful, elegant approach. You, or another child might do this. If this is done at the beginning, however, they will simply imitate the method and not appreciate why it was needed.	• *Now I'm going to try this problem myself, thinking aloud.* • *I might make some mistakes here - try to spot them for me.* • *This is one way of improving the solution.*
Sourced from: Module 3 'Problem Solving Lessons'. Developed by the Shell Center team at the Center for Research in Mathematics Education, University of Nottingham. *A small number of adaptations have been made to the source material to adjust for suitability for use by Early Years practitioners and Primary teachers.*	

Analyse and Reflect 1

Tasks as presented in instructional activities and plans

When you are assessing the appropriateness of tasks, considering the purpose or goals of a mathematical activity is important. We do not intend to suggest that tasks with lower-levels of cognitive challenge should be avoided completely. These tasks are necessary for procedural fluency. However, these tasks present limited opportunities to develop conceptual understanding and higher-order thinking.

Children must have opportunities to engage with cognitively challenging tasks in order to develop problem-solving skills and reasoning.

You could use Stein and Smith's (1998) framework for analysing tasks to support your reflection on and analysis of mathematical tasks. To develop your understanding of tasks and to think about your own teaching practice, you may like to do the following:

- The NRICH website contains many examples of problem-solving activities. Explore this website to find examples of tasks with higher-levels of cognitive demand. Note there is a section for both Early Years and Primary School.
- If your school uses a textbook, pick five pages at random and use Stein and Smith's (1998) framework for analysing tasks to categorise the mathematical exercises. Is there a balance across different task types or is there a tendency to favour a particular type of task?
- Consider the nature of the tasks in your most-used digital resources for mathematics teaching (e.g., websites, computer games, smartboard files, programming activities). Is there a balance across different task types or is there a tendency to favour a particular type of task?
- Use the overview provided in Stein and Smith's (1998) framework to review the mathematics tasks you have taught in the last week and consider whether incorporating a greater variety of task types might be of benefit to the children in your class.
- Consider the tasks in the activity plans on this website and categorise them using the Stein and Smith's approach.

Analyse and Reflect 2

Tasks as implemented by teachers and enacted by children (Tasks focus)

View the available videos to investigate how tasks were set up by teachers and implemented by children.

Before you do this, it is important that you have read the guidelines for Learning from Video.

We have also created a number of prompts based on the Teaching for Robust Understanding (TRU) observation guide for mathematics. The particular prompts we have chosen to focus on target the 'mathematics,' 'cognitive demand' and 'equitable access to content' dimensions of the TRU framework. The TRU observation guides also emphasise the importance for teachers of making connections across lessons.

Note: This is missing from our prompts and video samples due to the nature of the recording process where only individual lessons were filmed.

These prompt questions might also be useful as you reflect on recent or memorable experiences of teaching mathematics and consider ways to develop your own practice.

<table>
<tr><td colspan="2">Prompts for thinking about the mathematics and cognitive demand of tasks</td></tr>
<tr><td>Child Lens</td><td>Teacher Lens</td></tr>
<tr><td>

Do the students engage with the activity in ways that support the development of conceptual understanding and/or problem solving strategies?
Do they participate meaningfully in the mathematical work of the class?
Do the students have opportunities for mathematical reasoning (explaining rather than just stating answers)?
Do students engage with challenging ideas? Do they show a willingness to work on demanding tasks even without direct teacher input/encouragement?
Are students comfortable sharing partial or incorrect work as part of a whole class discussion?

</td><td>

Does the teacher use tasks and activities that provide multiple entry points and support multiple approaches to the mathematics content (i.e., not just one right answer or way of doing things)?
Does the teacher highlight important ideas and provide opportunities for students to engage with them?
Does the teacher support the purposeful use of mathematical language and of representations (e.g. use of manipulatives, tables, diagrams, symbols)?
Does the teacher position students as sense makers who can make sense of key conceptual ideas?
Does the teacher monitor students' engagement and adjust materials and activities to offer an appropriate level of challenge (i.e., support students without removing the challenge-productive struggle)?
Does the teacher support students in seeing mathematics as being coherent, connected and comprehensible?

</td></tr>
<tr><td colspan="2">General questions:
Each activity will have a specific mathematical goal but more generally, the mathematical goal can be understood as orchestrating opportunities for all students to work on core mathematical issues in ways that enable them to develop conceptual understandings, develop reasoning and problem solving skills, and use mathematical concepts, tools, methods and representations in relevant contexts.
Was this goal met? If so, how?

In relation to cognitive demand, the goal can be understood as orchestrating opportunities for all students to make their own sense of important mathematical ideas, developing deeper understandings by building on what they know. Was this goal met? If so, how?</td></tr>
<tr><td colspan="2">Adapted from:
Schoenfeld, A. H., and the Teaching for Robust Understanding Project. (2016). The Teaching for Robust Understanding (TRU) observation guide for mathematics: A tool for teachers, coaches, administrators, and professional learning communities. Berkeley, CA: Graduate School of Education, University of California, Berkeley. Retrieved from: http://map.mathshell.org/</td></tr>
</table>

Key ideas about tasks

- The nature of tasks selected for use with children is central to the type of mathematical thinking they have opportunities to engage in. Tasks should be chosen with attention to the mathematical content covered but also to the mathematical processes that will be used to find a solution (i.e. connecting, communicating, reasoning, argumentation, justifying, representing, problem-solving, and generalising)
- Cognitively-challenging tasks, where the solution method is not clear from the outset, offer opportunities for students to engage in genuine problem-solving activities. Such tasks may be represented in multiple ways (e.g. diagrams, manipulatives, symbols, or problem contexts) and students must engage with the underlying concepts to complete the task successfully.
- Good tasks are ones which are connected to the interests, prior knowledge, or previous experience of children. They are pitched just beyond current levels of understanding so that the solution is achievable with some effort.
- When working with challenging tasks, teachers must strike a balance between providing support where it is needed and retaining as much mathematical challenge as possible for children. It is recommended to allow children time to understand and engage with the problem; offer strategic rather than technical hints; encourage children to consider alternative methods and approaches. When children have had time to engage with the activity and develop some possible solutions of their own, it may be appropriate to model thinking and demonstrate conventional methods and make connections to the children’s ideas.
- There are a number of meaningful contexts within which mathematical understanding might be developed. Some of the most appropriate contexts for young children’s mathematical learning include play and playful approaches, the use of picture books and project work.