A Problem-Solving Approach to the Teaching of Number

Patrick Neary & Siún Nic Mhuirí

Maths4All

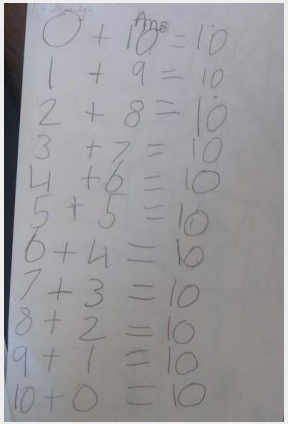
DCU Ollscoil Chathair Bhaile Átha Cliath Dublin City University

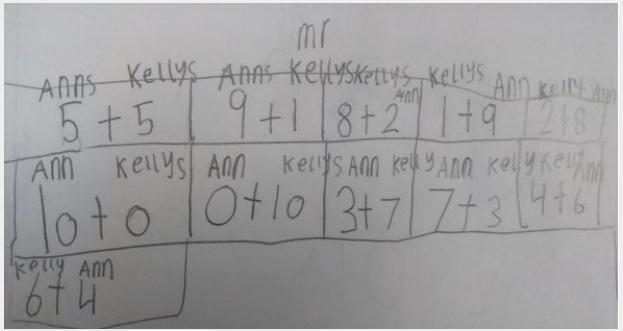
CASTeL

Science Foundation Ireland SFI For what's next

Outline

1. The Problem-Solving Approach
2. Introduction to Number Sense
3. Teaching an Unfamiliar Topic (multi-digit division) through Problem Solving
4. Enacting problem-solving approaches in your classroom





Section 1

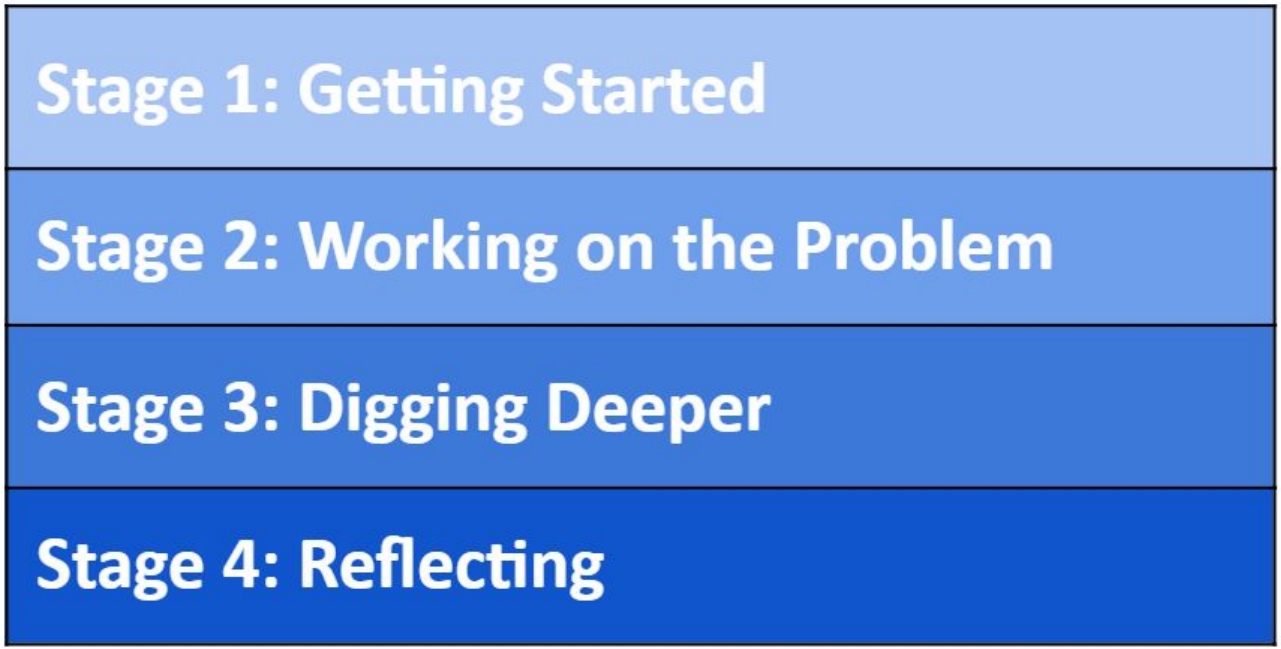
A problem-solving approach to the teaching of Number

The Problem-Solving Process

Video: First Class

Advantages of using a problem-solving approach

The Problem-Solving Process



Number: First Class

The school principal needs some help. She is expecting a delivery of ten packages tomorrow. She can stack some of the boxes in her office and some in the secretary's office.

The principal wants to know how many different ways she can share the boxes between her office and the secretary's office. She thinks there are about six different ways, but teacher thinks there are more than that.

As you watch:

1. How would you describe the general atmosphere of the lesson?
2. What are the children learning?

How would you describe the general atmosphere of the lesson?

What are the children learning?

Post your answer in the chat box.

Advantages to using a problem-solving approach

The Content	Cognitive Demand	Equitable Access to Content	Agency, Authority and Identity	Formative Assessment
Flexible & Connected Conceptually Focused **Productive Disposition**	**Productive Struggle** Grapple with Sense Making	Low Threshold High Ceiling **Every Child Actively Participating** 'Air Time'	Ownership over the Content Development of Positive Identities as Thinker and Learners	Build on Productive Beginnings Address Emerging Misunderstandings Meet children where they are.

Adapted from map.mathshell.org

Section 2

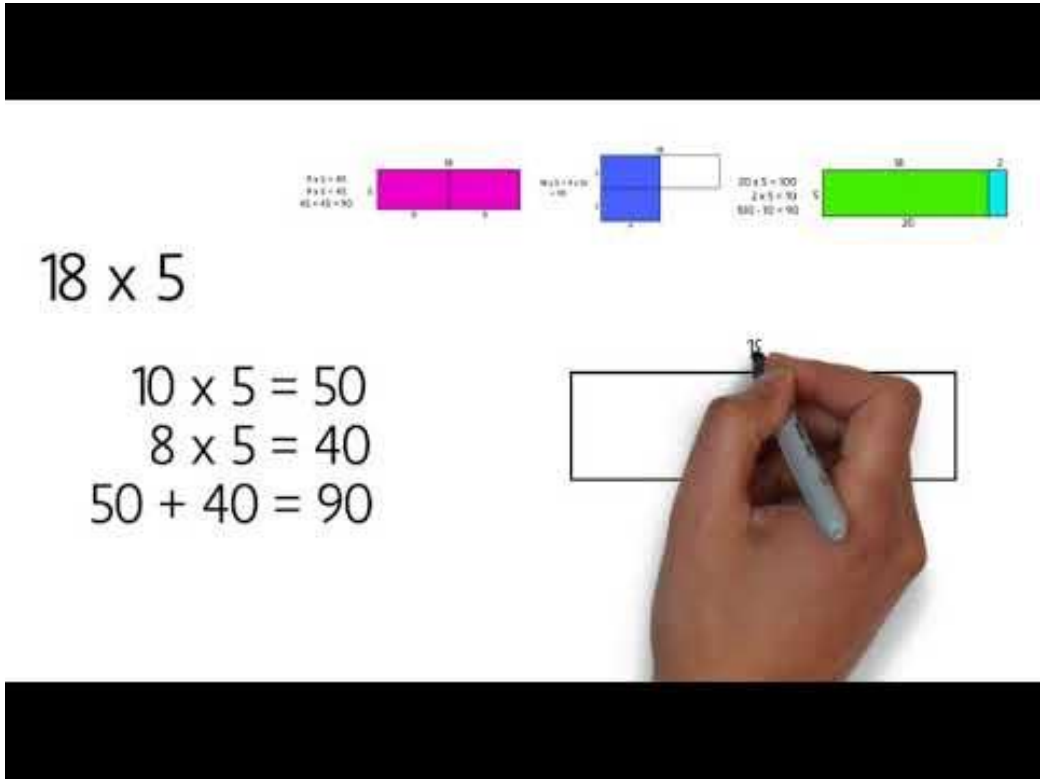
Introduction to Number Sense

Why do we teach tables?

What are the children learning?

Post your answer in the chat box.

What is number sense?



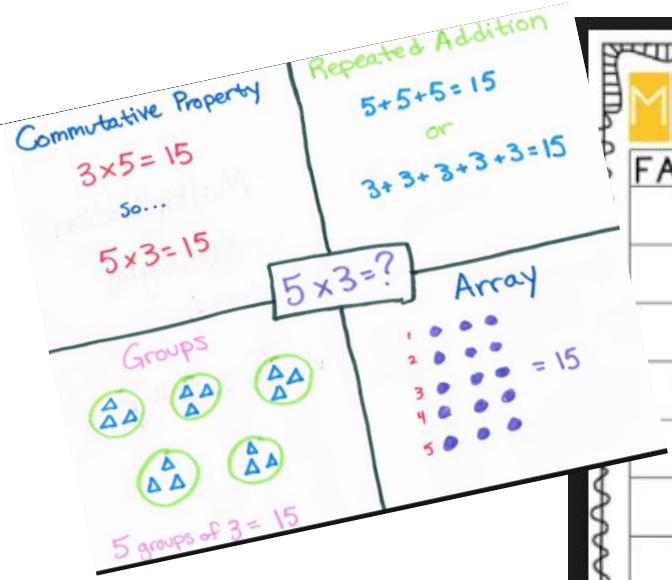
Traditional Teaching of Multiplication Tables

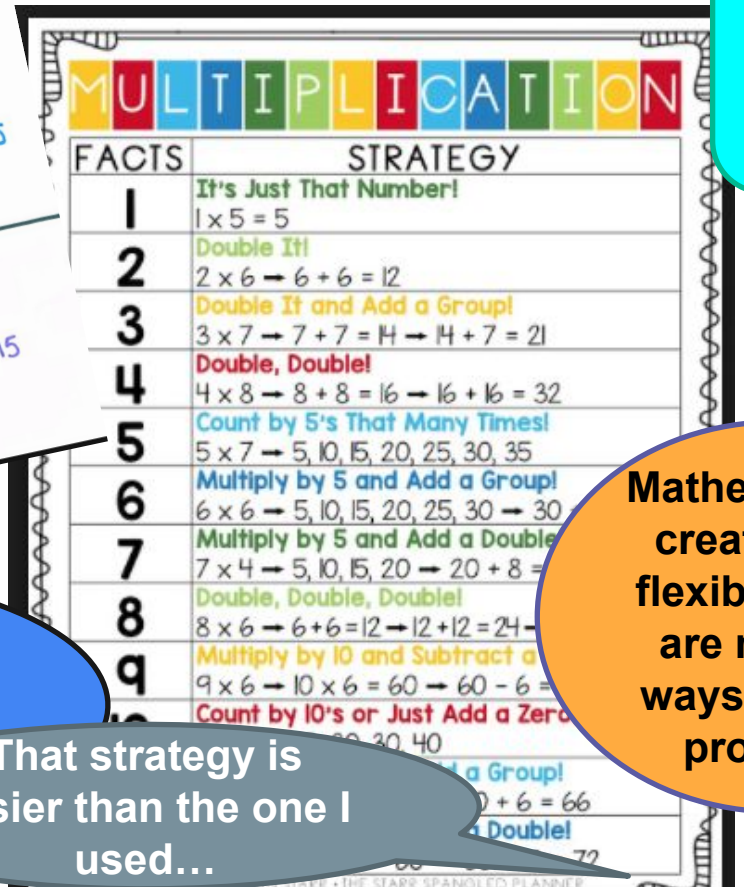
Classroom practices that emphasize speed, correctness/errors, formal tables tests

Mathematics is rigid, fixed, black and white. There is one method, one answer

To be good at Maths, I must be quick. I must memorise and use the teacher's methods

Newer approaches to Teaching Multiplication Tables





Classroom practices that emphasize multiple methods of representation and solution, self-assessment and meta-cognition

I can invent new ways of doing mathematics. To be good at mathematics, I should try to improve on my current strategies.

Mathematics is creative and flexible. There are multiple ways to solve problems

For this example, I'd use this strategy because

That strategy is easier than the one I used…

Advantages to using a number-sense approach

The Content	Cognitive Demand	Equitable Access to Content	Agency, Authority and Identity	Formative Assessment
Flexible & Connected Conceptually Focused Productive Disposition	Productive Struggle **Grapple with Sense Making**	**Low Threshold High Ceiling** Every Child Actively Participating 'Air Time'	Ownership over the Content **Development of Positive Identities as Thinker and Learners**	Build on Productive Beginnings Address Emerging Misunderstandings **Meet children where they are**

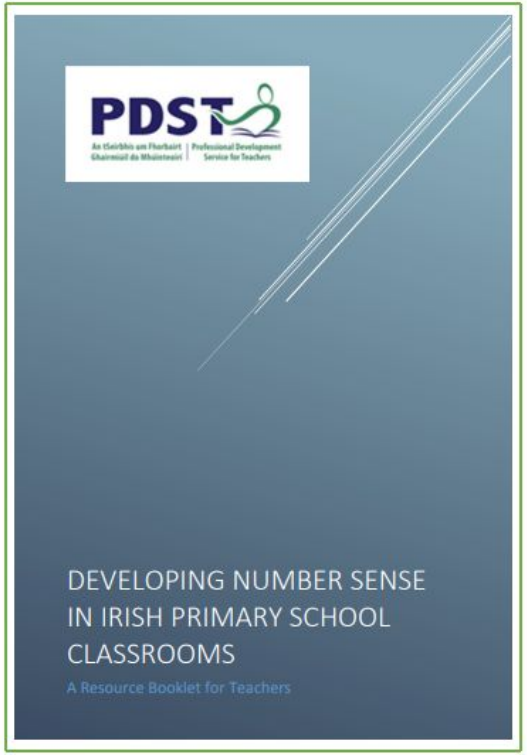
Adapted from map.mathshell.org

What is number sense?

The components of number sense have been defined as

- composing and decomposing numbers
- recognising the relative magnitude of numbers
- using benchmarks
- linking representations
- understanding the effects of operations
- inventing strategies
- estimating
- possessing a disposition toward making sense of numbers

(PDST, Developing Number Sense in Irish Primary Classrooms)



Number Talk (see Youcubed.org)

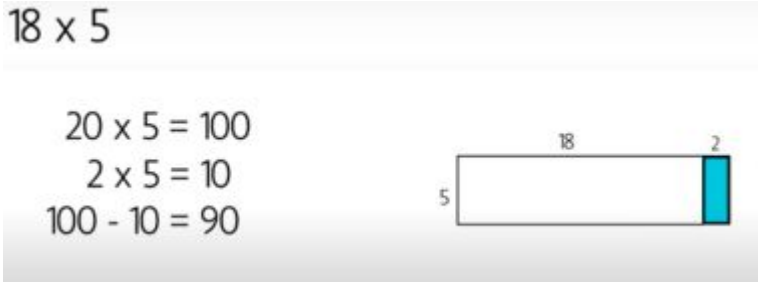
Launch	Explain that children will see a number problem and should try to work it out without paper or pencil. They will be asked to share their answer and to describe how they completed the calculation.
Explore	Show the number problem, e.g., 18 x 5. Ask students to quietly place their thumb at chest level to signal when they have come up with at least one solution.
Discuss	Children share their thinking. At first answers are put on the board. Children explain their reasoning and may opt to take incorrect answers from the board as the discussion goes on. The teacher should check to make sure that he/she is representing the child's thinking accurately on the board.
Extend	Choose one strategy and illustrate it with a sketch. Ask children to pick a different strategy and make a visual representation of it. Alternatively, have students solve a new problem using one of the strategies shared.
Reflect	Did you learn anything you did not know before?

A note on Boaler's use of diagrams

These diagrams ***reflect*** the children's ideas rather than ***direct*** them.

Teacher draws the diagram or writes notes to make these ideas visible and accessible to the rest of the class.

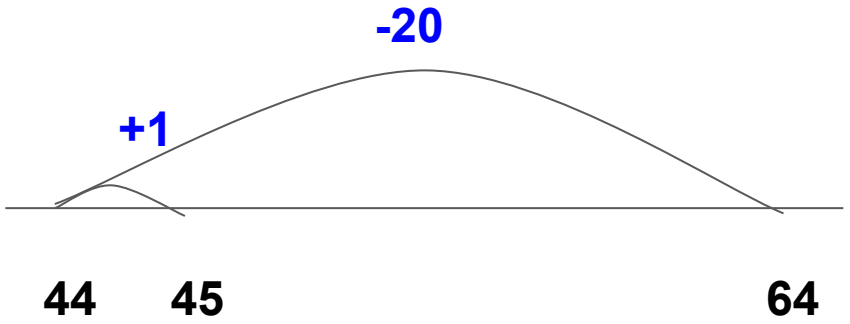
"I did 20 times 5 then took away 10"



Models to reflect rather than direct children's thinking

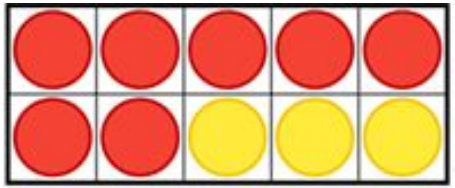
64 - 19

First I did 64 take away 20 to get 44 and I added one back on to get 45.



Current Practice

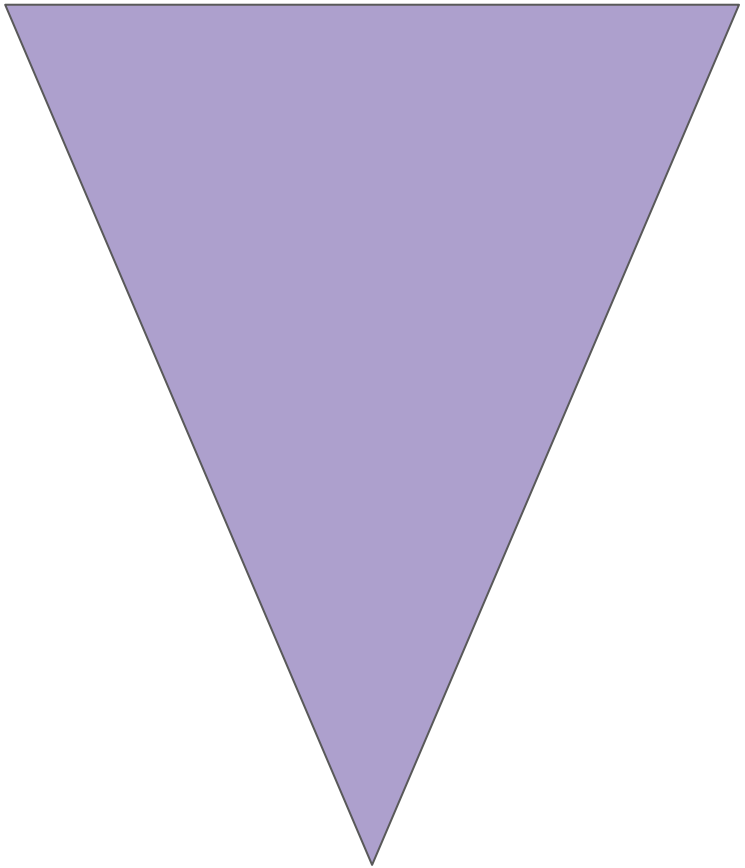
Approaches that encourage representations in early number work



3	12	4	10x5+4+1
5	8	1	55 (10 +1) x 5
10	2	6	4 x 12 +3 +4

Approaches that encourage sensemaking and multiple strategies in learning number facts (e.g., Number Talks, Target Boards)

Junior Primary



Senior Primary

Narrowing of representations used, and range of methods accepted?

Section 3

Division

Teaching an Unfamiliar Topic (multi-digit operation) through Problem Solving

The Role of Number Sense

Division by a
One-Digit Number
Understanding Remainders
3 Lessons

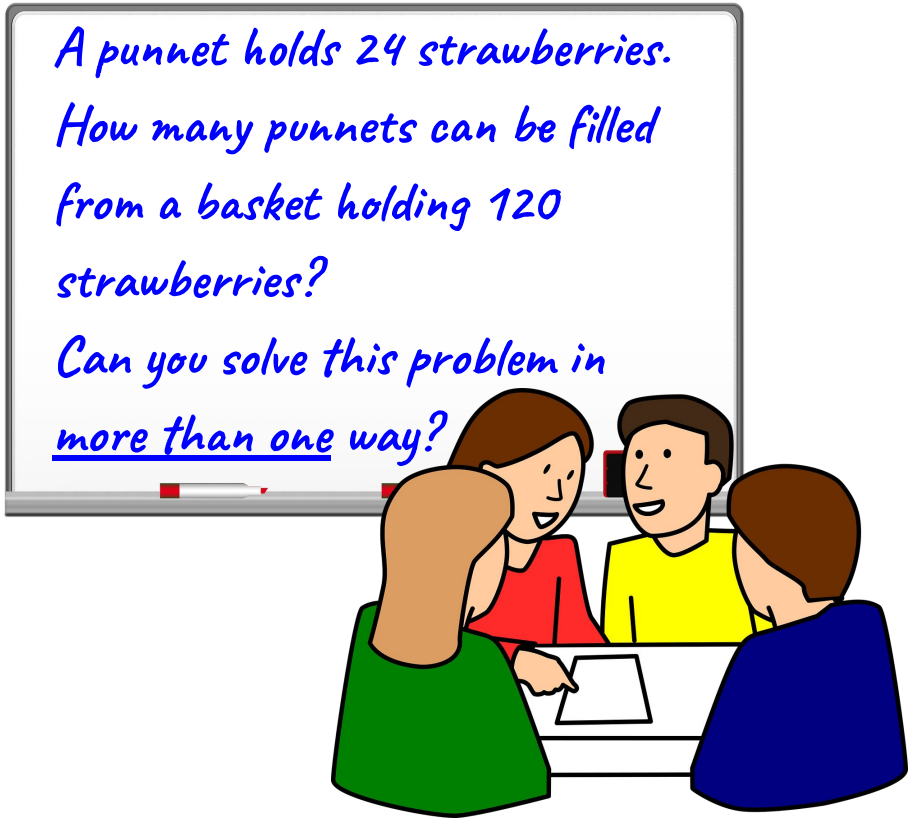
Division by a
Two-Digit Number
Using Invented Methods
3 Lessons

Division by a
Two-Digit Number
Introducing the Algorithm
3 Lessons

Unit Overview

Lesson 3: Multi-Digit Division

The child will use invented methods to divide a three-digit number by a two-digit number, to share a quantity into groups of a specific size.



How many different ways can you find to solve this problem?

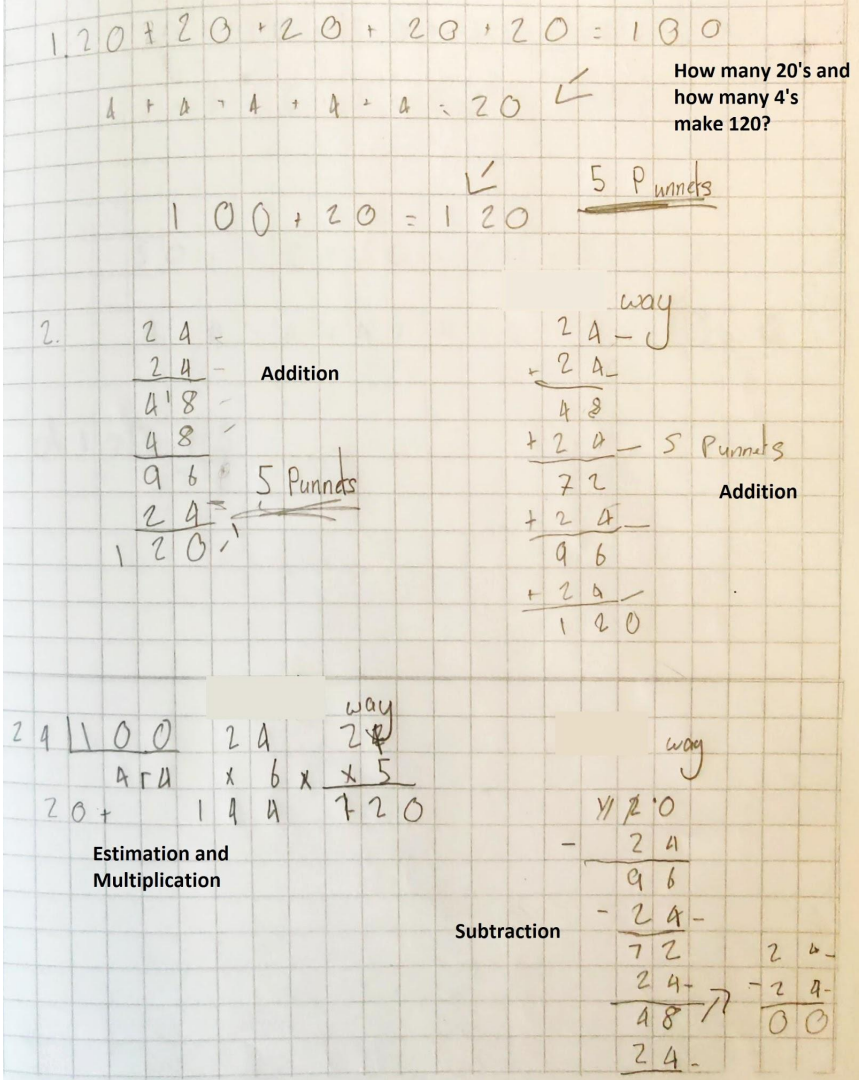
Did anyone at your group solve it in a similar/different way?

Whose method would you like to try next?

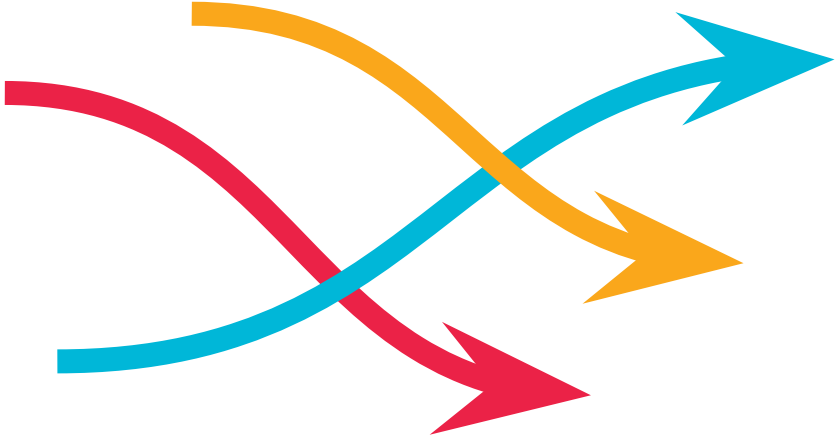
What have you discovered? How did you find that out?

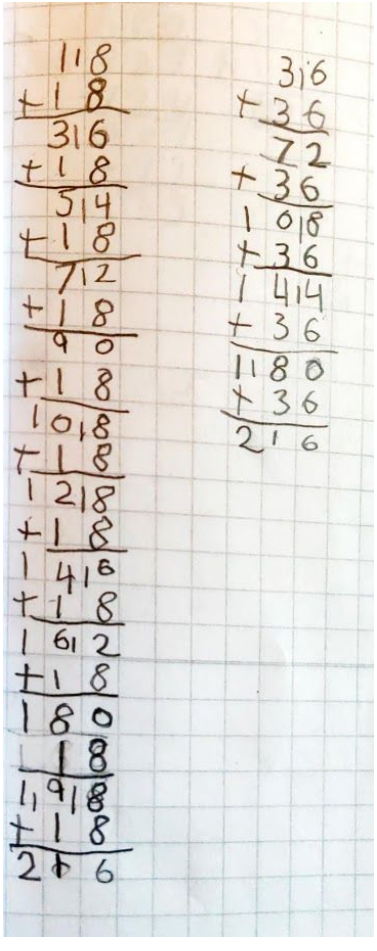
One child's work

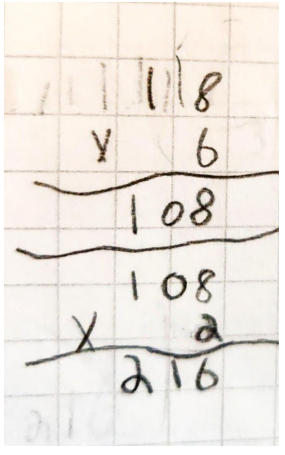
A punnet holds 24 strawberries.
How many punnets can be filled from a basket holding 120 strawberries?
Can you solve this problem in more than one way?

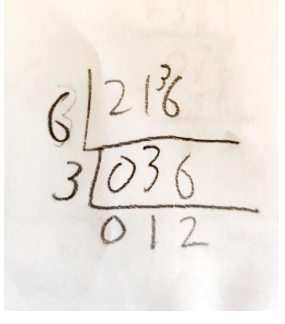


There were many different trajectories of learning









A car transporter delivered 216 cars over 18 trips. How many cars were carried on each trip?

We recorded **11** different ways of solving this problem!

What does success look like?

			5 1
6	)	3	1 1
		- 3	0
			1 1
			- 6
			5

			2	2 2
4	)	8	8	8
		-	8	
			0	8
			-	8
				0

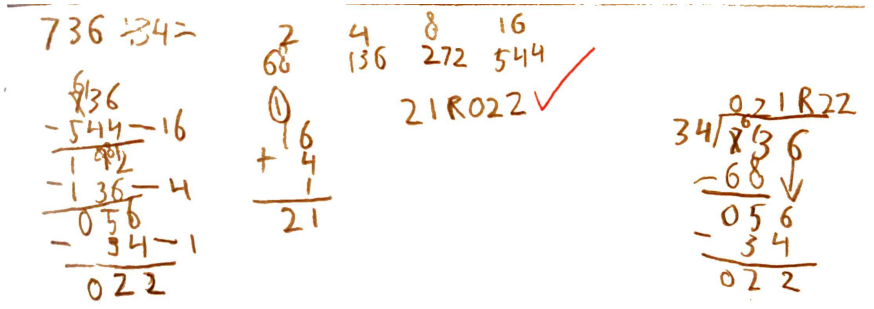
			1	3 5
5	)	6	7	6
		-	5	
			1	7
		-	1	5
				2 6
			-	2 5
				1

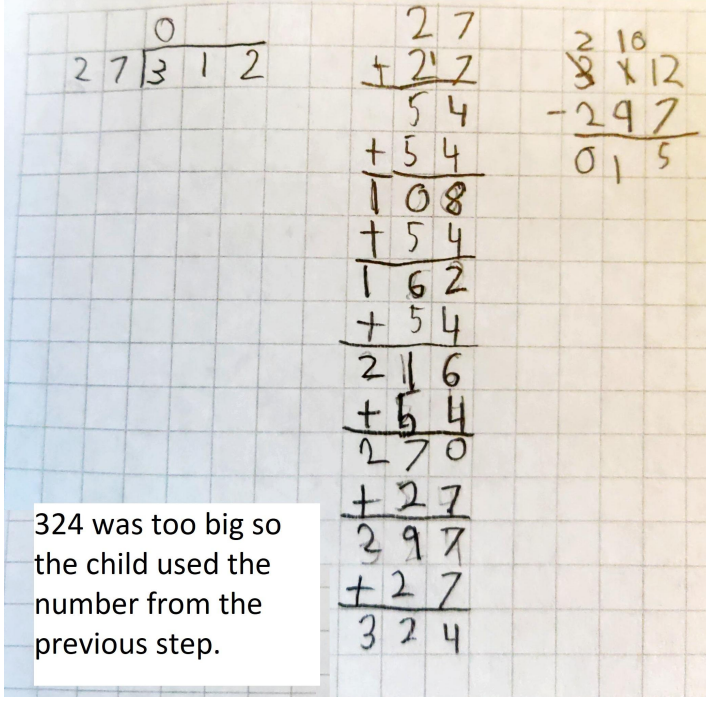
			1	3 9
4	)	5	5	6
		-	4	
			1	5
		-	1	2
				3 6
			-	3 6
				0

			1	8 7
3	)	5	6	3
		-	3	
			2	6
		-	2	4
				2 3
			-	2 1
				2

			1	4 4
3	)	4	3	4
		-	3	
			1	3
		-	1	2
				1 4
			-	1 2
				2

or





324 was too big so the child used the number from the previous step.

The children were asked to reflect on each lesson

Today I learned that if I am dividing by 3 (example) the remainder has to be less than 3.

Today I learned that remainders have patterns.

Today I did division using subtraction. I used James' way. It gives you the remainder, if there is any. Tomorrow I will double the divisor to make it quicker.

Reflections

Tomorrow I will use the long division way because it is interesting.

I will use the long division way and the subtraction way. They are easy and fun.

My favourite way is Michael's way. You multiply the divisor by an estimated number. It is quicker than adding.

I would double the number (not the big one) and then add on and on until we get an answer.

What do you think would be the greatest challenge to teaching a new topic in this way?

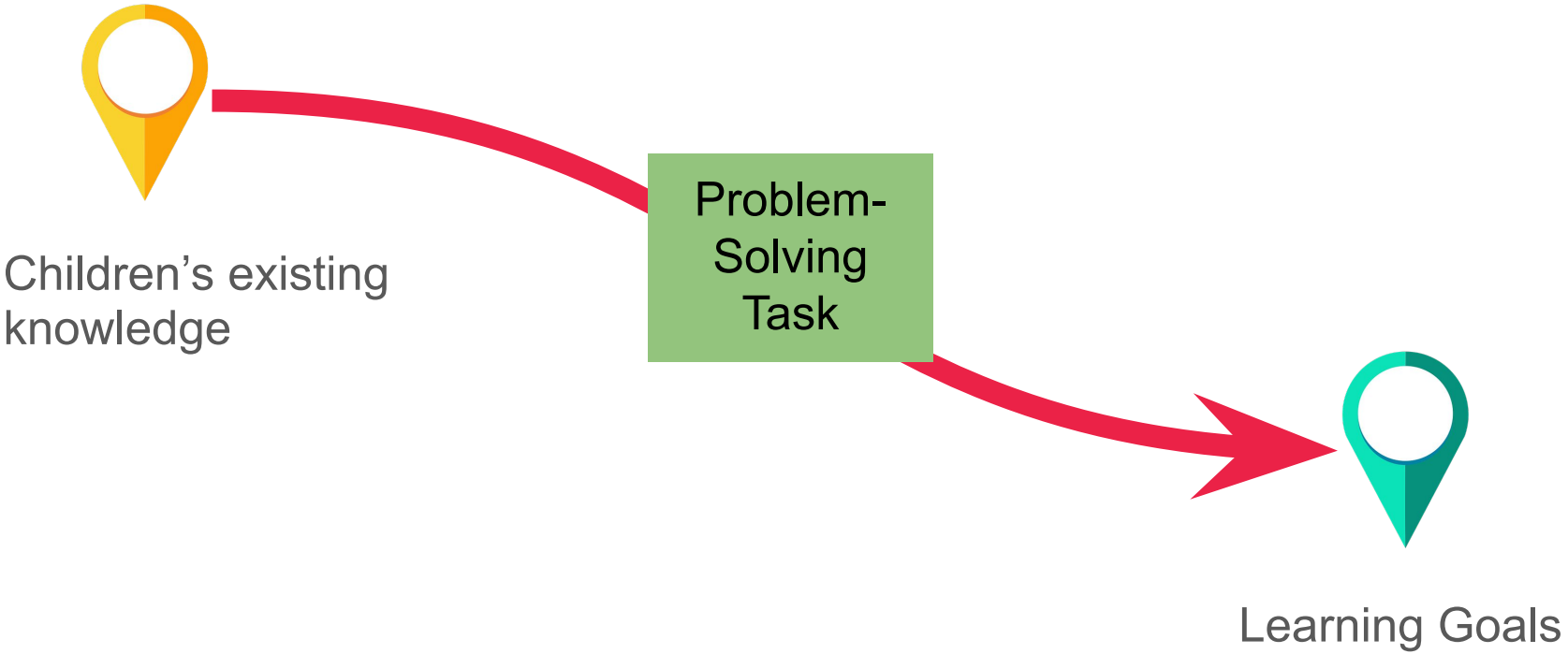
Post your answer in the chat box.

Section 4

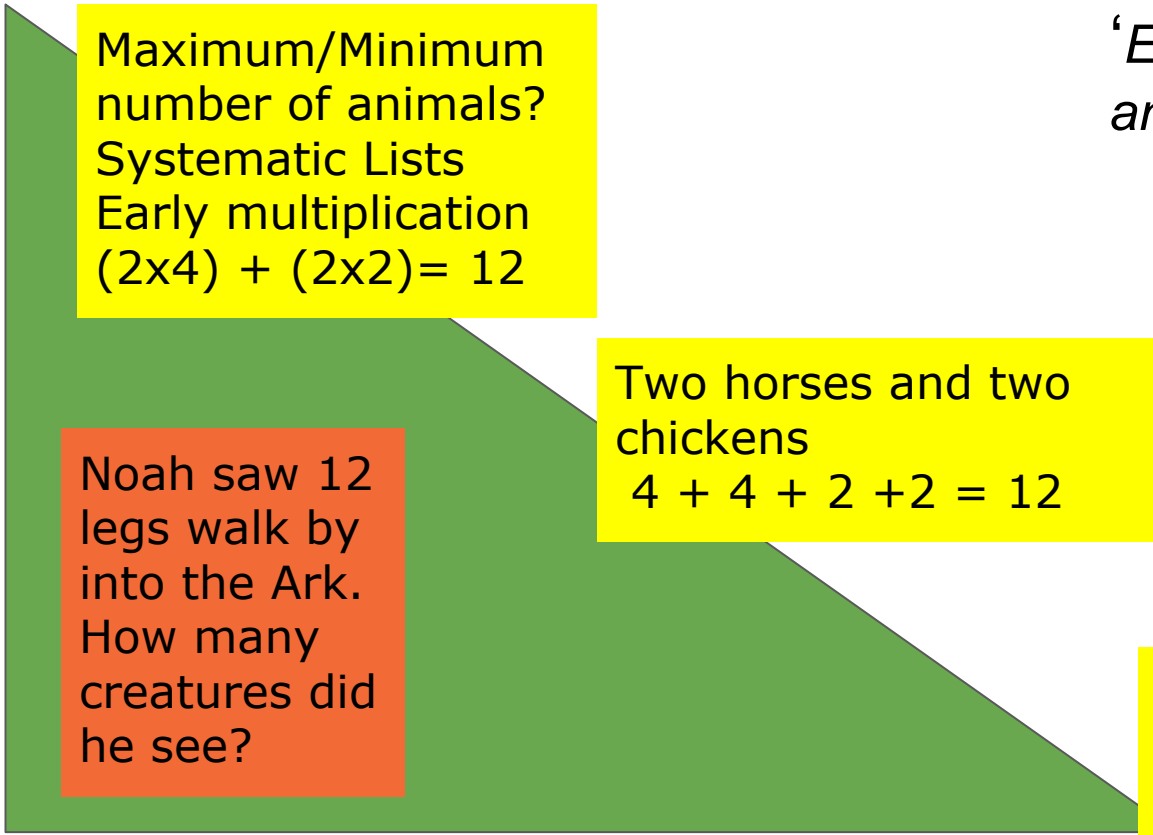
Enacting problem-solving approaches in your classroom

- Low threshold, high ceiling tasks
- Teacher's role in lessons

Choosing Tasks



Low Threshold-High Ceiling Tasks (Nrich 8769)



Maximum/Minimum number of animals?
Systematic Lists
Early multiplication
(2x4) + (2x2)= 12

Two horses and two chickens
4 + 4 + 2 +2 = 12

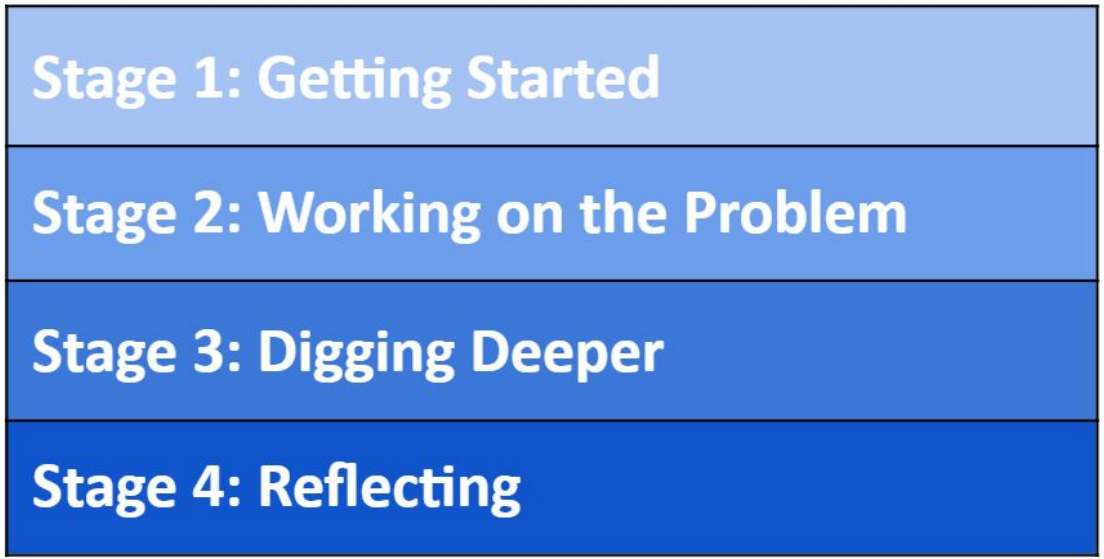
Noah saw 12 legs walk by into the Ark. How many creatures did he see?

Three horses
4 + 4 + 4 = 12

'Everyone can get started and everyone can get stuck'

Use addition and subtraction to solve real world situations and problems, making use of a range of strategies (Operations, Stage 2, Draft specification)

The Problem-Solving Process

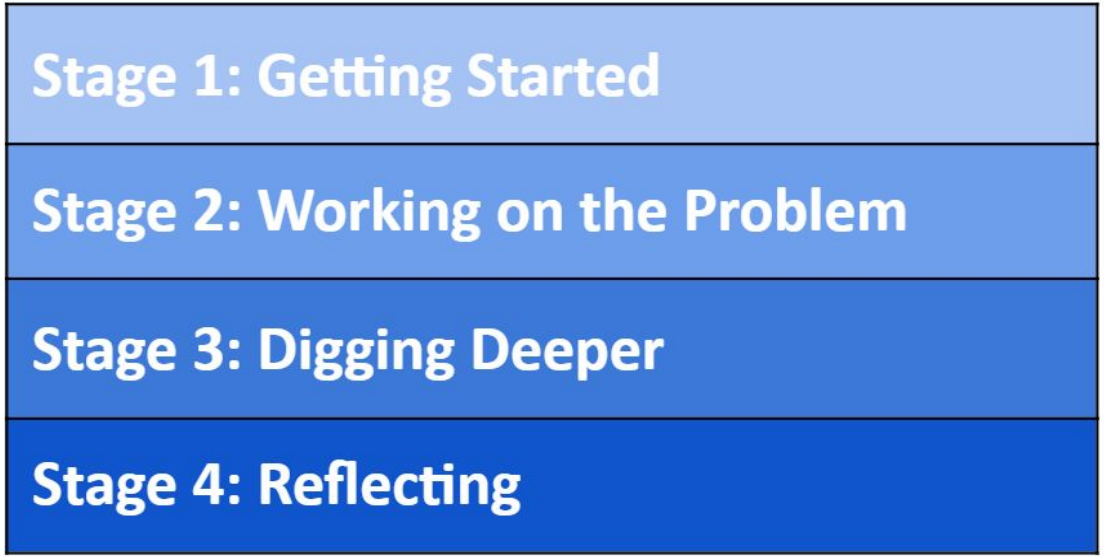


The Problem-Solving Process

Allow children time to understand and engage with the problem

What do you know?

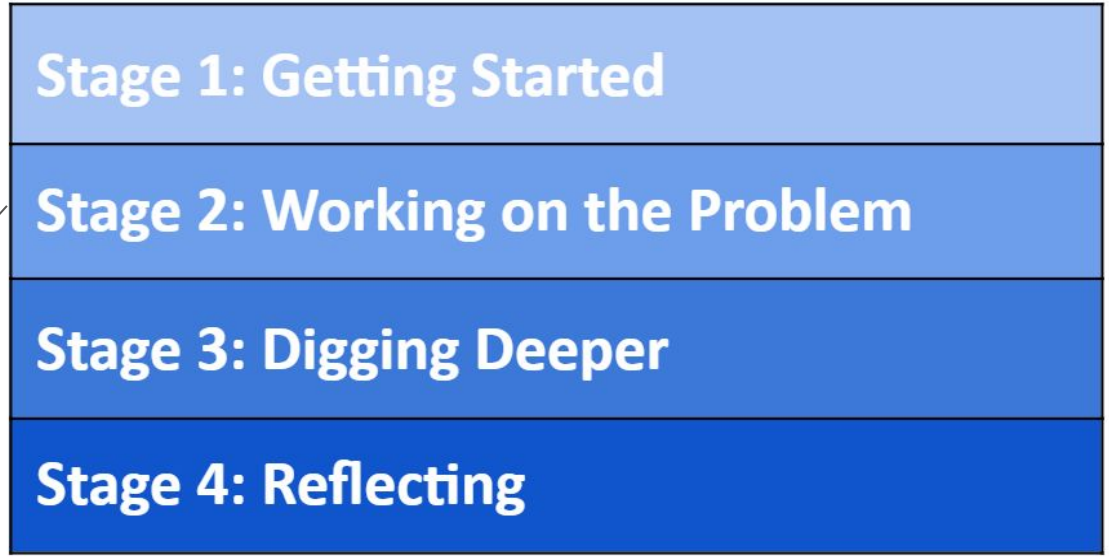
What are you trying to do?



The Problem-Solving Process

Offer strategic rather than technical hints. Encourage students to consider alternative methods and approaches

How could you get started on this problem? Is there another way of doing this?

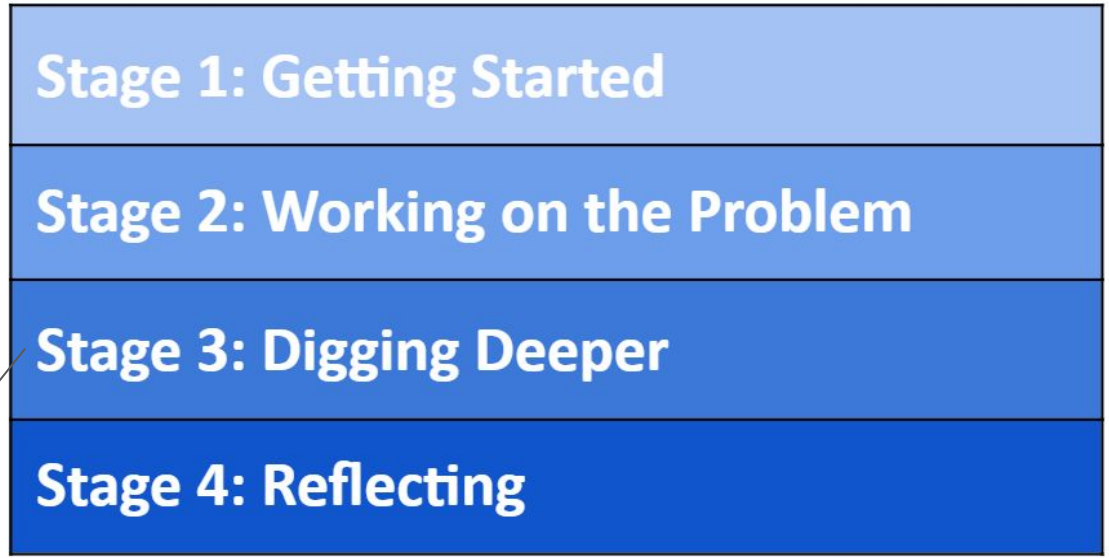


The Problem-Solving Process

Encourage explanation. Encourage students to consider the methods of others.

Can you explain your method?

Can you explain Sara's method in your own words?

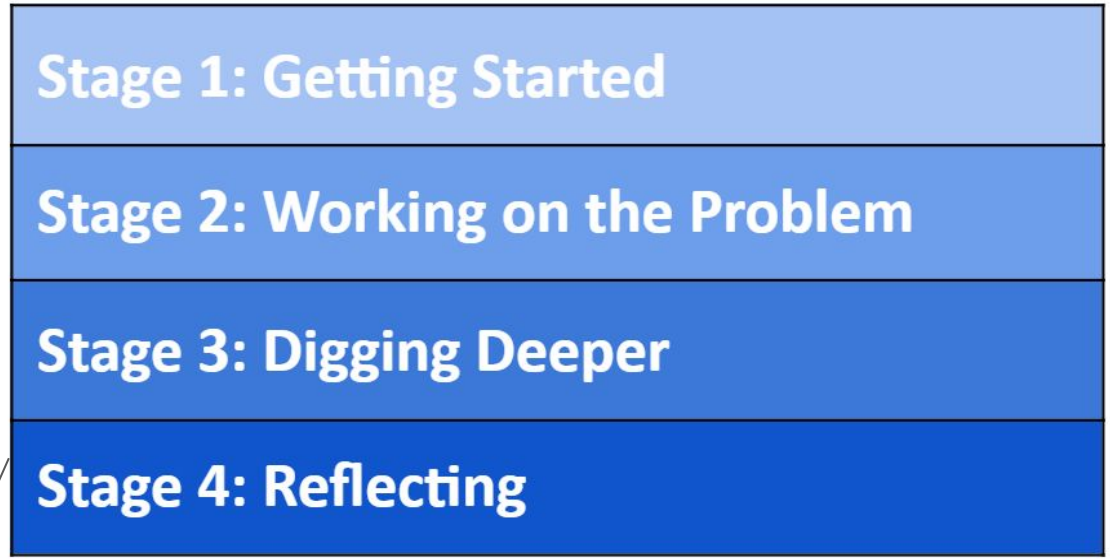


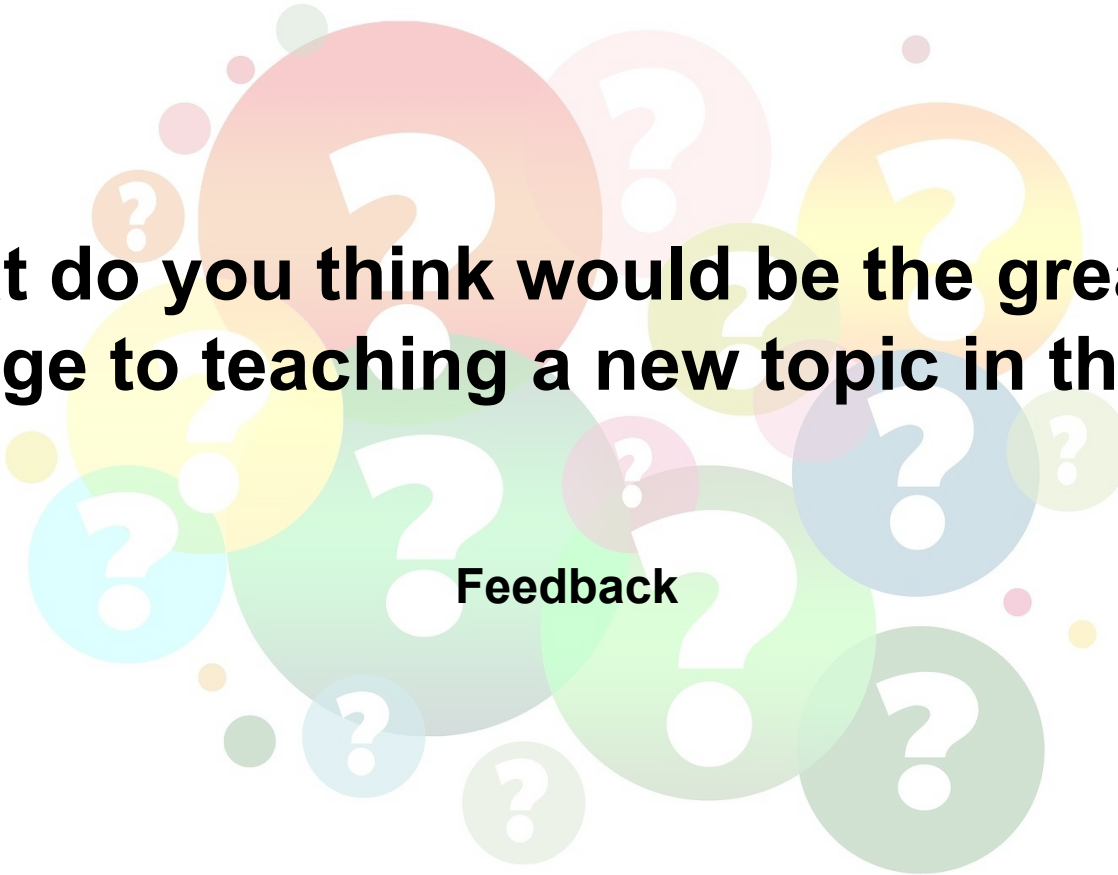
The Problem-Solving Process

Model thinking and powerful methods

I'm going to draw a picture showing Sean's method

Now I will try this problem myself, thinking aloud.

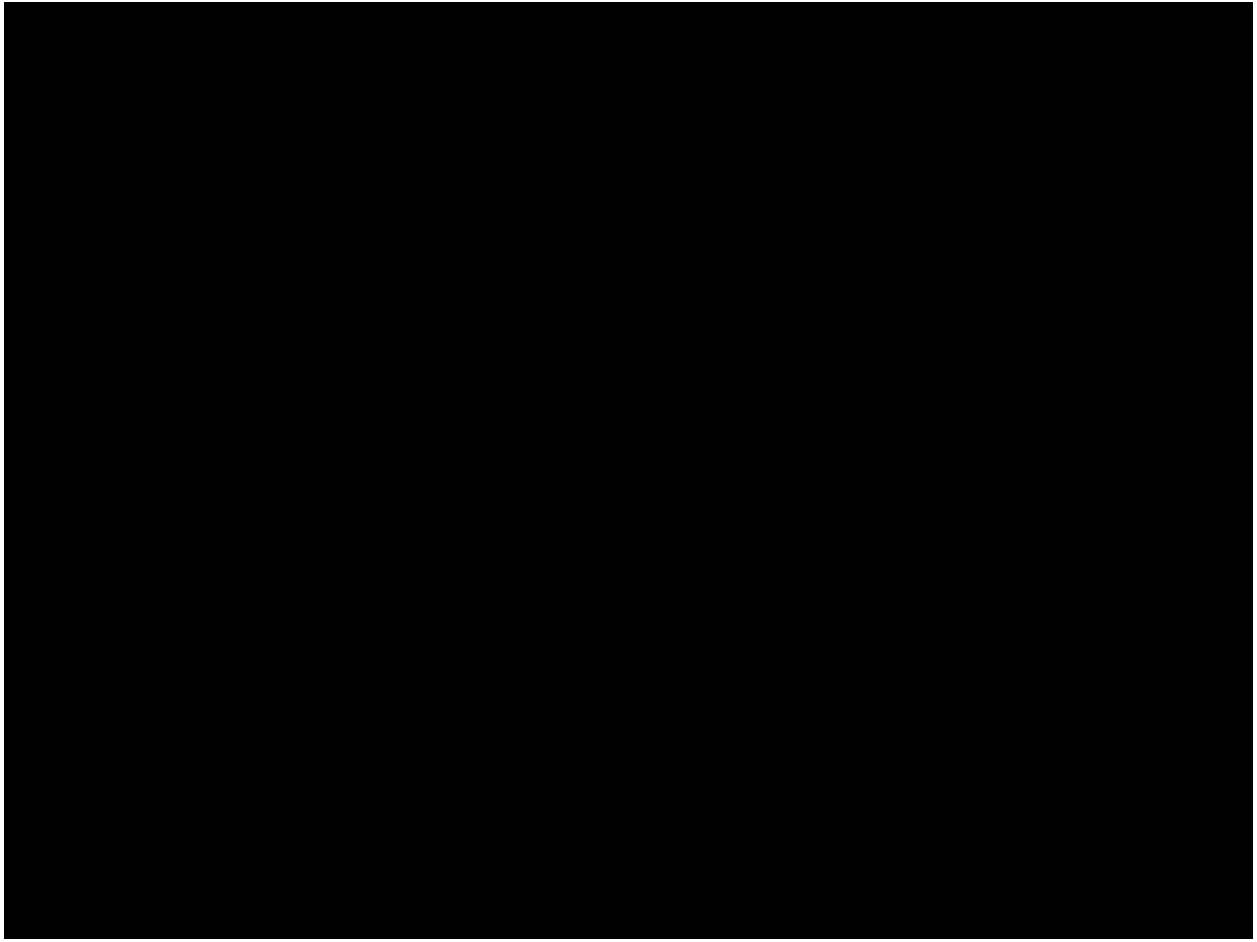




What do you think would be the greatest challenge to teaching a new topic in this way?

Feedback

Finding tasks on Nrich.maths.org



NRICH Tasks to Try

1st Class - 136 - *Noah*

2nd Class - 10586 - *Five Steps to 50*

3rd Class - 1177 - *Sealed Solution*

4th Class - 1130 - *Reach 100*

5th Class - 1047 - *Twenty Divided Into Six*

6th Class - 2127 - *A mixed up Clock*

Class Level indicated as a guide.
All of these problems are suitable for more than one class level.

Each problem includes 4 key elements for the teacher

Useful Resources

Jo Boaler YouCubed.org Number Sense Resources; Number Talk Example

PDST Number Sense Resources; Mental Maths Resources

Nrich Developing Early Number Sense

TRU Math Overview; Practical Advice for Teaching Problem Solving

Other sources for Tasks

NzMaths.co.nz

https://robertkaplinsky.com/prbl-search-engine/

https://learningfromchildren.org/3-act-tasks/ (K-2/Junior Primary Focus)