
Exploring Division with 5th Class

— A Problem-Solving Approach —

Patrick Neary

What the curriculum says:

- divide a three-digit number by a two-digit number, without and with a calculator

explore the concept of division with concrete materials

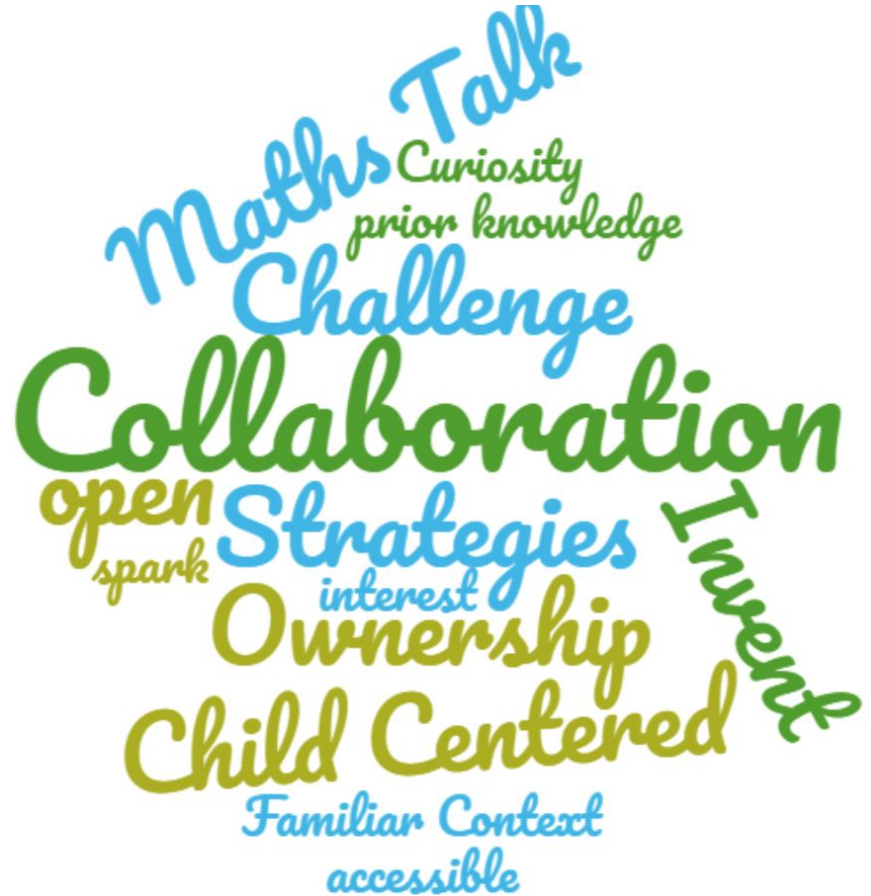
develop the long division algorithm from repeated subtraction and multiples of repeated subtraction

Initial Problem

A jar contains 225 marbles.
How many bags of 75 can be
filled from this jar?

The numbers chosen matter.

My aim was for the children to actively
engage in mathematical thinking.



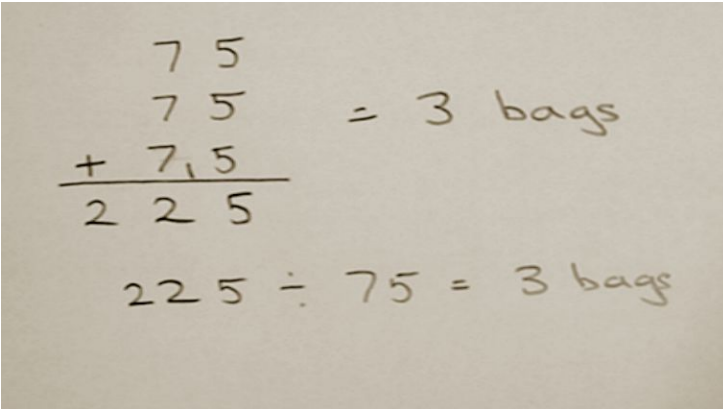
A jar contains 225 marbles. How many bags of 75 can be filled from this jar?

Children used their prior knowledge to approach this problem.

This child used a repeated addition strategy.

Others employed other methods e.g. repeated extraction.

Whole-class discussion focused on the samples of the children's work.



Handwritten mathematical work showing two methods for solving the problem:

$$\begin{array}{r} 75 \\ 75 \\ + 75 \\ \hline 225 \end{array} = 3 \text{ bags}$$
$$225 \div 75 = 3 \text{ bags}$$

(Reconstruction of child's work)

Exploring Multiple Solution Strategies

- Whole-Class Discussion
- Children's ideas and methods were prioritised
- Children's voices were prioritised
- Children were encouraged to 'try out' other methods
- Context impacted the suitability of different strategies

A second problem

$$240 \div 68$$

Method A is the same method the child used previously.

Method B is an alternative method that arose during whole-class discussion.

The child identified that method B told them how many marbles were left over i.e. a remainder. They used this method for the next problem.

The image shows three handwritten methods for solving the division problem $240 \div 68$.

Method A: Shows the addition of three 68s to reach 204, with the remainder 4. The calculation is written as:
$$\begin{array}{r} 68 \\ + 68 \\ + 68 \\ \hline 204 \end{array} - 3 \text{ bags}$$

Method B: Shows a subtraction-based method where 68 is subtracted from 240 in three steps, resulting in a remainder of 36. The calculation is written as:
$$\begin{array}{r} 240 \\ - 68 \\ \hline 172 \\ - 68 \\ \hline 104 \\ - 68 \\ \hline 36 \end{array} - 1 +$$

3 r 36

Method C: Shows a subtraction-based method where 86 is subtracted from 240 in three steps, resulting in a remainder of 16. The calculation is written as:
$$\begin{array}{r} 240 \\ - 86 \\ \hline 154 \\ - 86 \\ \hline 68 \\ - 86 \\ \hline 16 \end{array}$$

Next to Method C, the remainder is written as $4 \text{ r } 16$.

Creating a need for efficiency

This group developed a more efficient strategy whilst working on the problem.

You may notice a calculation error here. The remainder should be 7 and not 27. This led to a discussion about accuracy.

The calculation error **did not** take away from the advancements in the children's understandings.

The image shows a series of handwritten long division problems on a piece of paper. The problems are arranged vertically. Each problem consists of a dividend, a divisor, and a remainder. The first problem is $359 \div 16 = 22 \text{ r } 7$. The second is $359 \div 16 = 22 \text{ r } 3$. The third is $2847 \div 16 = 177 \text{ r } 7$. The fourth is $283 \div 64 = 4 \text{ r } 4$. The fifth is $119 \div 64 = 1 \text{ r } 55$. The sixth is $891 \div 64 = 13 \text{ r } 27$. The final line says "answer 23 r 27".

$$\begin{array}{r} 3 \overset{6}{\cancel{5}} 9 \\ - 16 \\ \hline 359 \\ - 16 \\ \hline 3 \overset{2}{\cancel{4}} 3 \\ - 16 \\ \hline 28 \overset{4}{\cancel{7}} \\ - 64 \\ \hline 2 \overset{7}{\cancel{8}} 3 \\ - 64 \\ \hline 1 \overset{1}{\cancel{2}} 19 \\ - 64 \\ \hline 0 \overset{5}{\cancel{1}} 55 \\ - 64 \\ \hline 8 \overset{9}{\cancel{1}} 1 \\ - 64 \\ \hline 27 \\ \text{answer } 23 \text{ r } 27 \end{array}$$

A focus on process rather than product

Method A: Some 'multiples of repeated subtraction' used.
Calculation error as $19 \times 4 = 76$ and not 72.

Method B: A more efficient method is developed for the same problem.

The child refined their solution strategy.

786 ÷ 19 = 41 r 7

A.

$\begin{array}{r} 786 \\ - 28 \\ \hline 758 \\ - 28 \\ \hline 730 \\ - 28 \\ \hline 702 \\ - 28 \\ \hline 674 \\ - 28 \\ \hline 646 \\ - 28 \\ \hline 618 \\ - 28 \\ \hline 590 \\ - 28 \\ \hline 562 \\ - 28 \\ \hline 534 \\ - 28 \\ \hline 506 \\ - 28 \\ \hline 478 \\ - 28 \\ \hline 450 \\ - 28 \\ \hline 422 \\ - 28 \\ \hline 394 \\ - 28 \\ \hline 366 \\ - 28 \\ \hline 338 \\ - 28 \\ \hline 310 \\ - 28 \\ \hline 282 \\ - 28 \\ \hline 254 \\ - 28 \\ \hline 226 \\ - 28 \\ \hline 198 \\ - 28 \\ \hline 170 \end{array}$	$\begin{array}{r} 786 \\ - 72 \times 10 \\ \hline 98 \\ - 72 \times 4 \\ \hline 26 \\ - 19 \times 5 \\ \hline 7 \end{array}$
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45 r 7 ?

B.

$\begin{array}{r} 786 \\ - 304 \times 10 \\ \hline 482 \\ - 304 \times 32 \\ \hline 178 \\ - 152 \times 40 \\ \hline 26 \\ - 19 \times 41 \\ \hline 7 \end{array}$
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41 r 7.

Student-Invented Method

This group used their understanding of doubles to support a repeated subtraction strategy.

The only teacher demonstration that occurred during this lesson was when the teacher revoiced the children's own thinking.

The image shows handwritten student work on a piece of paper. At the top, there is a multiplication table with two rows of numbers:

1	2	4	8	16
24	48	96	192	384

Below the table, there are two subtraction problems. The first one is:

$$\begin{array}{r} 629 \\ - 384 \\ \hline 245 \\ - 192 \\ \hline 133 \\ - 96 \\ \hline 47 \\ - 24 \\ \hline 23 \end{array}$$

The second one is:

$$\begin{array}{r} 16 \\ 8 \\ 4 \\ 1 \\ \hline 29 \end{array} \quad \text{r } 23$$

Next Steps

Concept

Lessons focused on building conceptual understanding.

Link

Lessons focused on connecting concepts and procedures.

Procedure

Lessons focused on the practise and application of procedures.

